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A Unified Framework for Analytical Transforms of the Y-Function

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Abstract: This paper explores the properties of the Y-function using various important integral transforms. We evaluate and derive clear expressions for the Euler, Hankel, Whittaker, K-, and P_δ-transforms applied to the Y-function kernel. Using the Mellin-Barnes integral representation, we show that the resulting images can be clearly expressed in terms of the original Y-function and the generalized Fox-H function. These findings create new functional relations and broaden the existing techniques for hypergeometric-type functions.

Keywords: Y-function;Euler Transform;Mellin Transform;Laplace Transform;Whittaker transform; Hankel Transform

I. INTRODUCTION

1.1 Y Function :

The Y-function of Yang [9] as follows(see also [2]):

$$Y_{p,q}^{m,n}\left[t;y;\alpha;\begin{matrix}\{a_p,A_p\}_1^p\\ \{b_q,B_q\}_1^q\end{matrix}\right]=\frac{1}{2\pi i}\int_L \Theta(s)t^{-s}e^{sy}s^{\alpha}ds \tag{1.1.1}$$

in which

$$\Theta(s)=\frac{A(s)B(s)}{C(s)D(s)} \tag{1.1.2}$$

where

$$A(s)=\prod_{j=1}^{m}\Gamma(b_j-B_js)$$

$$B(s)=\prod_{j=1}^{n}\Gamma(1-a_j+A_js)$$

$$C(s)=\prod_{j=m+1}^{q}\Gamma(1-b_j+B_js)$$

$$D(s)=\prod_{j=n+1}^{p}\Gamma(a_j-A_js)$$

with $x\in\mathbb{C},y\in\mathbb{C},z\in\mathbb{R},q\geq 1,0\leq n\leq p,1\leq m\leq q,\{a_j,b_j\}\in\mathbb{C},\{a_j,b_j\}\in\mathbb{R}_+$

The pole of the Y-function is

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$$k_\lambda^l = \frac{b_j + \lambda}{B_j} \quad (j = 1, n; \lambda \in \mathbb{N} \cup \{0\}) \tag{1.1.3}$$

and

$$k_\lambda^l = \frac{1 - a_j + \lambda}{A_j} \quad (j = 1, n; \lambda \in \mathbb{N} \cup \{0\}) \tag{1.1.4}$$

- When $y = \alpha = 0$ in (1.1.1),the Y-function is reduced to Fox H-Function

$$Y_{p,q}^{m,n}\left[t; 0; 0; \begin{matrix} \{a_p, A_p\}_1^p \\ \{b_q, B_q\}_1^q \end{matrix}\right] = H_{p,q}^{m,n}\left[\frac{1}{t}; \begin{matrix} \{a_p, A_p\}_1^p \\ \{b_q, B_q\}_1^q \end{matrix}\right] \tag{1.1.5}$$

1.2 Beta Function :

The Beta function is defined as follows[1]

$$\beta(m, n) = \int_0^\infty (1 - x)^{n-1} x^{m-1} dx \tag{1.2.1}$$

where $m, n > 0$

Relation with the Gamma function:

the beta function can be written in the form of gamma function as follows:

$$\beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m + n)} \tag{1.2.2}$$

1.3 Integral Transforms:

1.3.1 Euler Transform:

The Euler transform [7] of a function f(z) is defined as:

$$B\{f(z); a, b\} = \int_0^1 z^{a-1}(1 - z)^{b-1} f(z) dz \tag{1.3.1}$$

a,b ∈ C, $R(a) > 0, R(b) > 0$

1.3.2 Whittaker Transform:

The Whittaker transform is determined by the integral formula, which involves the Whittaker function and is defined as [8].

$$\int_0^\infty e^{-\frac{t}{2}} t^{\delta-1} W_{\alpha,\beta}(t) dt = \frac{\Gamma(\frac{1}{2} + \beta + \delta)\Gamma(\frac{1}{2} - \beta + \delta)}{\Gamma(1 - \alpha + \delta)} \tag{1.3.2}$$

where $R(\beta \pm \delta) > -\frac{1}{2}$ and $W_{\alpha,\beta}(t)$ is the whittaker confluent hypergeometric function.

1.3.3 Hankel Transform:

The definition of the Hankel transform of f(x) , represented by $H_v\{f(x)\}$,is[[5]](eqn. 2.43)

$$g(p; v) = \int_0^\infty (px)^{\frac{1}{2}} J_v(px) f(x) dx; \qquad p > 0$$

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(1.3.3)

$J_v(px)$ is the Bessal function of first kind of order v.

The integral in problem (1.4.1) can be solved using the following formula ([5],p.56-57,eqn.2.46)

$$\int_0^\infty x^{\lambda-1} J_v(ax)dx = 2^{\lambda-1} a^{-\lambda} \frac{\Gamma(\frac{\lambda+v}{2})}{\Gamma(1+\frac{v-\lambda}{2})}$$

(1.3.4)

1.3.4 K-Transform:

The K-Transform is defined by the integral equation [6] (see also [3])

$$\mathbb{R}_v[f(x);p] = g[p;v] = \int_0^\infty (px)^{\frac{1}{2}} K_v(px) f(x)dx$$

(1.3.5)

Here $R(v) > 0; K_v(x)$ is the second kind of Bessel function which is defined in ([6])

$$K_v(z) = \left(\frac{\pi}{2z}\right)^{\frac{1}{2}} W_{0,v}(2z)$$

where $W_{0,v}(.)$ is the Whittaker function defined as

$$W_{\beta,\zeta}(z) = \frac{\Gamma(-2\beta)}{\Gamma\left(\frac{1}{2}-\alpha-\beta\right)} M_{\alpha,\beta}(z) + \frac{\Gamma(2\beta)}{\left(\frac{1}{2}+\alpha+\beta\right)} M_{\alpha,-\beta}(z)$$

In evaluating the integrals,the following result given by Mathai et al.[5] will be used

$$\int_0^\infty t^{\rho-1} K_v(ax)dx = 2^{\rho-2} a^{-\rho} \Gamma\left(\frac{\rho \pm v}{2}\right); \qquad \mathbb{R}_v(a) > 0; \mathbb{R}_v(\rho \pm v) > 0$$

(1.3.6)

1.3.5 $\boldsymbol{P_\delta}$-Transform:

The $P_\delta[f(t);s]$ transform of a function $f(t)$ is defined as [4] (see also[3])

$$P_\delta[f(t);s] = F_{p_\delta}(s) = \int_0^\infty [1+(\delta-1)s]^{-\frac{t}{\delta-1}} f(t)dt; \qquad \delta > 1$$

(1.3.7)

P_δ-Transform of the power function $t^{\beta-1}$ is given by

$$P_\delta[t^{\beta-1};s] = \left(\frac{\delta-1}{ln[1+(\delta-1)]s}\right)^\beta \Gamma(\beta)$$

(1.3.8)

II. RESULTS AND PROOFS

Theorem1:(Euler-Tranform)
For $x, y \in \mathbb{C}, z \in \mathbb{R}, q \geq 1, 0 \leq n \leq p, 1 \leq m \leq q, \{a_j, b_j\} \in \mathbb{C}, \{a_j, b_j\} \in \mathbb{R}_+$ and a,b ∈ C, $R(a) > 0, R(b) > 0$ then

$$B\left\{\left[Y_{p,q}^{m,n}\left[t;y;\alpha;\begin{matrix}\{a_p,A_p\}_1^p \\ \{b_q,B_q\}_1^q\end{matrix}\right]\right](x)\right\} = \Gamma(b)\, Y_{p+1,q+1}^{m,n+1}\left[1;y;\alpha;\begin{matrix}\{a_p,A_p\}_1^p;(1-a,1) \\ \{b_q,B_q\}_1^q;(1-a-b,1)\end{matrix}\right]$$

(2.0.1)

Proof:

Apply (1.3.1) on (1.1.1)

$$= B\left\{\left[Y_{p,q}^{m,n}\left[t; y; \alpha; \begin{matrix} \{a_p, A_p\}_1^p \\ \{b_q, B_q\}_1^q \end{matrix}\right]\right]; a, b\right\}$$

$$= B\left\{\left(\frac{1}{2\pi i}\int_L \Theta(s) t^{-s} e^{sy} s^{\alpha} ds\right); a, b\right\}$$

$$= \frac{1}{2\pi i}\int_L \Theta(s) t^{-s} e^{sy} s^{\alpha} ds \int_0^1 t^{a-1}(1-t)^{b-1} dt$$

$$= \frac{1}{2\pi i}\int_L \Theta(s) e^{sy} s^{\alpha} ds \int_0^1 t^{a-s-1}(1-t)^{b-1} dt$$

Using (1.2.2),we get

$$= \frac{1}{2\pi i}\int_L \Theta(s) e^{sy} s^{\alpha} ds \frac{\Gamma(a-s)\Gamma(b)}{\Gamma(a-s+b)}$$

$$= \Gamma(b)\, Y_{p+1,q+1}^{m,n+1}\left[1; y; \alpha; \begin{matrix} \{a_p, A_p\}_1^p; (1-a, 1) \\ \{b_q, B_q\}_1^q; (1-a-b, 1) \end{matrix}\right] \tag{2.0.2}$$

Corollary 1:

Using (1.1.5),we get

$$\Gamma(b)\, Y_{p+1,q+1}^{m,n+1}\left[1; y; \alpha; \begin{matrix} \{a_p, A_p\}_1^p; (1-a, 1) \\ \{b_q, B_q\}_1^q; (1-a-b, 1) \end{matrix}\right] = \Gamma(b)\, H_{p+1,q+1}^{m,n+1}\left[1; \begin{matrix} \{a_p, A_p\}_1^p; (1-a, 1) \\ \{b_q, B_q\}_1^q; (1-a-b, 1) \end{matrix}\right] \tag{2.0.3}$$

Theorem 2:(Whittaker Transform)

For $x, y \in \mathbb{C}, z \in \mathbb{R}, q \geq 1, 0 \leq n \leq p, 1 \leq m \leq q, \{a_j, b_j\} \in \mathbb{C}, \{a_j, b_j\} \in \mathbb{R}_+$ and $R(\beta \pm \delta) > -\frac{1}{2}$ then

$$W_{\alpha,\beta}\left\{\left[Y_{p,q}^{m,n}\left[t; y; \alpha; \begin{matrix} \{a_p, A_p\}_1^p \\ \{b_q, B_q\}_1^q \end{matrix}\right]\right](t)\right\} = \Gamma(b) Y_{p+2,q+1}^{m,n+2}\left[1; y; \alpha; \begin{matrix} \{a_p, A_p\}_1^p; \left(\frac{1}{2}-\beta-\delta, 1\right); \left(\frac{1}{2}+\beta-\delta, 1\right) \\ \{b_q, B_q\}_1^q; (\alpha-\delta, 1) \end{matrix}\right] \tag{2.0.4}$$

Proof:

Apply (1.3.2) on (1.1.1)

$$= \int_0^{\infty} e^{-\frac{t}{2}} t^{\delta-1} W_{\alpha,\beta}(t)\left\{\left[Y_{p,q}^{m,n}\left[t; y; \alpha; \begin{matrix} \{a_p, A_p\}_1^p \\ \{b_q, B_q\}_1^q \end{matrix}\right]\right]\right\} dt$$

$$= \int_0^{\infty} e^{-\frac{t}{2}} t^{\delta-1} W_{\alpha,\beta}(t)\left\{\left(\frac{1}{2\pi i}\int_L \Theta(s) t^{-s} e^{sy} s^{\alpha} ds\right)\right\} dt$$

$$= \frac{1}{2\pi i}\int_L \Theta(s) e^{sy} s^{\alpha} ds \int_0^{\infty} e^{-\frac{t}{2}} t^{\delta-s-1} W_{\alpha,\beta}(t) dt$$

Using(1.3.2),we get

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$$= \frac{1}{2\pi i}\int_L \Theta(s)e^{sy}s^{\alpha}ds\,\frac{\Gamma\left(\frac{1}{2}+\beta+\delta-s\right)\Gamma\left(\frac{1}{2}-\beta+\delta-s\right)}{\Gamma(1-\alpha+\delta-s)}$$

$$= \Gamma(b)\, Y_{p+2,q+1}^{m,n+2}\left[1;y;\alpha;\begin{matrix}\{a_p,A_p\}_1^p;\left(\frac{1}{2}-\beta-\delta,1\right);\left(\frac{1}{2}+\beta-\delta,1\right)\\ \{b_q,B_q\}_1^q;(\alpha-\delta,1)\end{matrix}\right] \tag{2.0.5}$$

Corollary 2 :

Using (1.1.5),we get

$$\Gamma(b)\, Y_{p+2,q+1}^{m,n+2}\left[1;y;\alpha;\begin{matrix}\{a_p,A_p\}_1^p;\left(\frac{1}{2}-\beta-\delta,1\right);\left(\frac{1}{2}+\beta-\delta,1\right)\\ \{b_q,B_q\}_1^q;(\alpha-\delta,1)\end{matrix}\right]$$
$$= \Gamma(b)\, H_{p+2,q+1}^{m,n+2}\left[1;\begin{matrix}\{a_p,A_p\}_1^p;\left(\frac{1}{2}-\beta-\delta,1\right);\left(\frac{1}{2}+\beta-\delta,1\right)\\ \{b_q,B_q\}_1^q;(\alpha-\delta,1)\end{matrix}\right] \tag{2.0.6}$$

Theorem 3:(Hankel Transform)

For $x,y\in\mathbb{C}, z\in\mathbb{R}, q\geq 1, 0\leq n\leq p, 1\leq m\leq q, \{a_j,b_j\}\in\mathbb{C}, \{a_j,b_j\}\in\mathbb{R}_+$ and $p>0$ then

$$H_v\left\{\left[Y_{p,q}^{m,n}\left[t;y;\alpha;\begin{matrix}\{a_p,A_p\}_1^p\\ \{b_q,B_q\}_1^q\end{matrix}\right]\right](t)\right\} = 2^{\frac{1}{2}}p^{-1}Y_{p+1,q+1}^{m,n+1}\left[\frac{2}{p};y;\alpha;\begin{matrix}\{a_p,A_p\}_1^p;\left(\frac{1}{4}-\frac{v}{2},\frac{1}{2}\right)\\ \{b_q,B_q\}_1^q;\left(\frac{3}{4}-\frac{v}{2},\frac{1}{2}\right)\end{matrix}\right] \tag{2.0.7}$$

Proof : Apply (1.3.3) on (1.1.1)

$$= \int_0^\infty (pt)^{\frac{1}{2}}J_v(pt)\left[Y_{p,q}^{m,n}\left[t;y;\alpha;\begin{matrix}\{a_p,A_p\}_1^p\\ \{b_q,B_q\}_1^q\end{matrix}\right]\right]dt$$

$$= \int_0^\infty (pt)^{\frac{1}{2}}J_v(pt)\left[\frac{1}{2\pi i}\int_L \Theta(s)t^{-s}e^{sy}s^{\alpha}ds\right]dt$$

$$= \frac{1}{2\pi i}\int_L \Theta(s)e^{sy}s^{\alpha}ds\left[\int_0^\infty (pt)^{\frac{1}{2}}J_v(pt)t^{-s}dt\right]$$

$$= \frac{1}{2\pi i}\int_L \Theta(s)e^{sy}s^{\alpha}ds\; p^{\frac{1}{2}}\int_0^\infty (t)^{\frac{1}{2}-s+1-1}J_v(pt)dt$$

Using 1.3.4,we get

$$= \frac{1}{2\pi i}\int_L \Theta(s)e^{sy}s^{\alpha}ds\; p^{\frac{1}{2}}2^{\frac{3}{2}-s-1}p^{-\frac{3}{2}+s}\frac{\Gamma\left(\frac{\frac{3}{2}-s+v}{2}\right)}{\Gamma\left(1+\frac{v-\frac{3}{2}+s}{2}\right)}$$

$$= \frac{1}{2\pi i}\int_L \Theta(s) e^{sy} s^{\alpha} ds\, 2^{\frac{1}{2}-s} p^{-\frac{3}{2}+s+\frac{1}{2}} \frac{\Gamma\left(\frac{\frac{3}{2}-s+v}{2}\right)}{\Gamma\left(\frac{v+\frac{1}{2}+s}{2}\right)}$$

$$= 2^{\frac{1}{2}} p^{-1} \frac{1}{2\pi i}\int_L \Theta(s) e^{sy} s^{\alpha} \left(\frac{2}{p}\right)^{-s} ds \frac{\Gamma\left(\frac{\frac{3}{2}-s+v}{2}\right)}{\Gamma\left(\frac{v+\frac{1}{2}+s}{2}\right)}$$

$$= 2^{\frac{1}{2}} p^{-1} Y_{p+1,q+1}^{m,n+1}\left[\frac{2}{p}; y; \alpha; \begin{matrix} \{a_p, A_p\}_1^p; \left(\frac{1}{4}-\frac{v}{2}, \frac{1}{2}\right) \\ \{b_q, B_q\}_1^q; \left(\frac{3}{4}-\frac{v}{2}, \frac{1}{2}\right) \end{matrix}\right] \tag{2.0.8}$$

Corollary 3 :

Using (1.1.5),we get

$$2^{\frac{1}{2}} p^{-1} Y_{p+1,q+1}^{m,n+1}\left[\frac{2}{p}; y; \alpha; \begin{matrix} \{a_p, A_p\}_1^p; \left(\frac{1}{4}-\frac{v}{2}, \frac{1}{2}\right) \\ \{b_q, B_q\}_1^q; \left(\frac{3}{4}-\frac{v}{2}, \frac{1}{2}\right) \end{matrix}\right] = 2^{\frac{1}{2}} p^{-1} H_{p+1,q+1}^{m,n+1}\left[\frac{p}{2}; \begin{matrix} \{a_p, A_p\}_1^p; \left(\frac{1}{4}-\frac{v}{2}, \frac{1}{2}\right) \\ \{b_q, B_q\}_1^q; \left(\frac{3}{4}-\frac{v}{2}, \frac{1}{2}\right) \end{matrix}\right] \tag{2.0.9}$$

Theorem 4:(K-Transform)

For $x, y \in \mathbb{C}, z \in \mathbb{R}, q \geq 1, 0 \leq n \leq p, 1 \leq m \leq q, \{a_j, b_j\} \in \mathbb{C}, \{a_j, b_j\} \in \mathbb{R}_+$ and $R(v) > 0$; and $\mathbb{R}_v(a) > 0$; $\mathbb{R}_v(\rho \pm v) > 0$ then

$$K_v\left\{\left[Y_{p,q}^{m,n}\left[t; y; \alpha; \begin{matrix} \{a_p, A_p\}_1^p \\ \{b_q, B_q\}_1^q \end{matrix}\right]\right](t)\right\} = 2^{-\frac{1}{2}} p^{-1} Y_{p+2,q}^{m,n+2}\left[\frac{2}{p}; y; \alpha; \begin{matrix} \{a_p, A_p\}_1^p; \left(\frac{1}{4}-\frac{v}{2}, \frac{1}{2}\right); \left(\frac{1}{4}+\frac{v}{2}, \frac{1}{2}\right) \\ \{b_q, B_q\}_1^q \end{matrix}\right] \tag{2.0.10}$$

Proof : Apply (1.3.5) on (1.1.1)

$$= \int_0^\infty (pt)^{\frac{1}{2}} K_v(pt) \left[Y_{p,q}^{m,n}\left[t; y; \alpha; \begin{matrix} \{a_p, A_p\}_1^p \\ \{b_q, B_q\}_1^q \end{matrix}\right]\right] dt$$

$$= \int_0^\infty (pt)^{\frac{1}{2}} K_v(pt) \left[\frac{1}{2\pi i}\int_L \Theta(s) t^{-s} e^{sy} s^{\alpha} ds\right] dt$$

$$= \frac{1}{2\pi i}\int_L \Theta(s) e^{sy} s^{\alpha} ds \left[\int_0^\infty (pt)^{\frac{1}{2}} K_v(pt) t^{-s} dt\right]$$

$$= \frac{1}{2\pi i}\int_L \Theta(s) e^{sy} s^{\alpha} ds\, p^{\frac{1}{2}} \int_0^\infty (t)^{\frac{1}{2}-s} J_v(pt) dt$$

Using 1.3.6,we get

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$$= \frac{1}{2\pi i}\int_L \Theta(s)e^{sy}s^{\alpha}ds\; p^{\frac{1}{2}}2^{-s-\frac{1}{2}}p^{s-\frac{3}{2}}\Gamma\left(\frac{\frac{3}{2}-s+v}{2}\right)\Gamma\left(\frac{\frac{3}{2}-s-v}{2}\right)$$

$$= \frac{1}{2\pi i}\int_L \Theta(s)e^{sy}s^{\alpha}ds 2^{-s-\frac{1}{2}}p^{s-1}\Gamma\left(\frac{\frac{3}{2}-s+v}{2}\right)\Gamma\left(\frac{\frac{3}{2}-s-v}{2}\right)$$

$$= \frac{1}{2\pi i}\int_L \Theta(s)e^{sy}s^{\alpha}ds 2^{-\frac{1}{2}}p^{-1}\left(\frac{2}{p}\right)^{-s}\Gamma\left(\frac{\frac{3}{2}-s+v}{2}\right)\Gamma\left(\frac{\frac{3}{2}-s-v}{2}\right)$$

$$= 2^{\frac{1}{2}}p^{-1}Y_{p+1,q+1}^{m,n+1}\left[\frac{2}{p};y;\alpha;\begin{matrix}\{a_p,A_p\}_1^p;\left(\frac{1}{4}-\frac{v}{2},\frac{1}{2}\right)\\ \{b_q,B_q\}_1^q;\left(\frac{3}{4}-\frac{v}{2},\frac{1}{2}\right)\end{matrix}\right] \tag{2.0.11}$$

Corollary 4 :

Using (1.1.5),we get

$$2^{\frac{1}{2}}p^{-1}Y_{p+1,q+1}^{m,n+1}\left[\frac{2}{p};y;\alpha;\begin{matrix}\{a_p,A_p\}_1^p;\left(\frac{1}{4}-\frac{v}{2},\frac{1}{2}\right)\\ \{b_q,B_q\}_1^q;\left(\frac{3}{4}-\frac{v}{2},\frac{1}{2}\right)\end{matrix}\right] = 2^{-\frac{1}{2}}p^{-1}Y_{p+2,q}^{m,n+2}\left[\frac{2}{p};y;\alpha;\begin{matrix}\{a_p,A_p\}_1^p;\left(\frac{1}{4}-\frac{v}{2},\frac{1}{2}\right);\left(\frac{1}{4}+\frac{v}{2},\frac{1}{2}\right)\\ \{b_q,B_q\}_1^q\end{matrix}\right] \tag{2.0.12}$$

Theorem 5:(P_δ-Transform)

For $x,y \in \mathbb{C}, z \in \mathbb{R}, q \geq 1, 0 \leq n \leq p, 1 \leq m \leq q, \{a_j,b_j\} \in \mathbb{C}, \{a_j,b_j\} \in \mathbb{R}_+$ and $\delta > 1$ then

$$P_\delta\left\{\left[Y_{p,q}^{m,n}\left[t;y;\alpha;\begin{matrix}\{a_p,A_p\}_1^p\\ \{b_q,B_q\}_1^q\end{matrix}\right]\right](t)\right\} = \frac{1}{\left(\frac{ln[1+(\delta-1)\rho]}{\delta-1}\right)}Y_{p+1,q}^{m,n+1}\left[\frac{1}{\left(\frac{ln[1+(\delta-1)\rho]}{\delta-1}\right)};y;\alpha;\begin{matrix}\{a_p,A_p\}_1^p;(0,1)\\ \{b_q,B_q\}_1^q\end{matrix}\right] \tag{2.0.13}$$

Proof : Apply (1.3.7) on (1.1.1)

$$= \int_0^{\infty}[1+(\delta-1)\rho]^{-\frac{t}{\delta-1}}\left[Y_{p,q}^{m,n}\left[t;y;\alpha;\begin{matrix}\{a_p,A_p\}_1^p\\ \{b_q,B_q\}_1^q\end{matrix}\right]\right]dt$$

$$= \int_0^{\infty}[1+(\delta-1)\rho]^{-\frac{t}{\delta-1}}\frac{1}{2\pi i}\int_L \Theta(s)t^{-s}e^{sy}s^{\alpha}ds\, dt$$

$$= \frac{1}{2\pi i}\int_L \Theta(s)e^{sy}s^{\alpha}ds\int_0^{\infty}[1+(\delta-1)\rho]^{-\frac{t}{\delta-1}}t^{-s}dt$$

Let $x = 1+(\delta-1)\rho$, then

$$= \frac{1}{2\pi i}\int_L \Theta(s)e^{sy}s^{\alpha}ds \int_0^{\infty} x^{-\frac{t}{\delta-1}}\, t^{-s}dt$$

$$= \frac{1}{2\pi i}\int_L \Theta(s)e^{sy}s^{\alpha}ds \int_0^{\infty} t^{-s}\, x^{-\frac{t}{\delta-1}ln(x)}dt$$

Using

$$\Gamma(z) = \int_0^{\infty} t^{z-1}e^{-at}dt$$

Here

$$z = 1 - s,$$

$$a = \frac{ln(x)}{\delta - 1}$$

Then the integral evaluate

$$\frac{\Gamma(z)}{a^z}$$

$$= \frac{1}{2\pi i}\int_L \Theta(s)e^{sy}s^{\alpha}ds \frac{\Gamma(1-s)}{\left(\frac{ln[1+(\delta-1)\rho]}{\delta-1}\right)^{1-s}}$$

$$= \frac{1}{\left(\frac{ln[1+(\delta-1)\rho]}{\delta-1}\right)} Y_{p+1,q}^{m,n+1}\left[\frac{1}{\left(\frac{ln[1+(\delta-1)\rho]}{\delta-1}\right)}; y; \alpha; \begin{matrix}\{a_p, A_p\}_1^p; (0,1) \\ \{b_q, B_q\}_1^q\end{matrix}\right]$$

(2.0.14)

Corollary 5 :

Using (1.1.5),we get

$$\frac{1}{\left(\frac{ln[1+(\delta-1)\rho]}{\delta-1}\right)} Y_{p+1,q}^{m,n+1}\left[\frac{1}{\left(\frac{ln[1+(\delta-1)\rho]}{\delta-1}\right)}; y; \alpha; \begin{matrix}\{a_p, A_p\}_1^p; (0,1) \\ \{b_q, B_q\}_1^q\end{matrix}\right]$$
$$= \frac{1}{\left(\frac{ln[1+(\delta-1)\rho]}{\delta-1}\right)} H_{p+1,q}^{m,n+1}\left[\frac{\delta-1}{(ln[1+(\delta-1)\rho])}; \begin{matrix}\{a_p, A_p\}_1^p; (0,1) \\ \{b_q, B_q\}_1^q\end{matrix}\right]$$

(2.0.15)

III. CONCLUSION

This study provides a clear framework for the key properties of the Y-function. We give closed-form expressions for several important transforms and show how they relate to the generalized Fox H-function. This offers a strong mathematical basis for future applications. These findings not only build on the existing theory of hypergeometric-type functions but also supply important analytical tools for solving complex fractional-order differential equations in physics and engineering.

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