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On Dual Equations with Four Unknowns $x - y * z = k * z^3$, $x * y = w^3$

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Abstract*:* This paper discusses on finding integer solutions to the system of double equations with four unknowns given by $x - y * z = k * z^3$, $x * y = w^3$. Four choices of integer solutions to the considered dual equations are presented. Employing the obtained integer solutions, the process of formulating Second order Ramanujan numbers is illustrated.

Keywords*:* Dual equations with four unknowns, System of double equations, Integer solutions, second order Ramanujan numbers

I. INTRODUCTION

It is well known that the Diophantine problems offer an unlimited field for research by reason of their variety. The successful completion of exhibiting all integers satisfying their requirements set forth in the problems add to further progress of number theory. Systems of indeterminate quadratic equations of the form $ax + c = a^2, bx + d = b^2$ where a, b, c, d are non-zero distinct constant, have been investigator for integer solutions by several authors and with a few possible exceptions, most of them where primarily concerned with rational solutions. Even those existing works, wherein integral solutions have been attempted, deal essentially with specific cases only and do not exhibit methods of finding integral solutions in the general form.

In [1-4], the following four systems of Double Equations are studied:

i. $x + y = z + w, y + z = (x + w)^2$

ii. $x + y = u^2, \frac{x}{D} + y = v^2$

iii. $N_1 - N_2 = k, N_1 * N_2 = k^3 s^2, k \geq 0$

iv. $x - y = u^2, \frac{x}{D} - y = v^2$

Also, in [5-9], the following four systems of Triple Equations are studied:

v. $x + y = 2a^2, 2x + y = 5a^2 + b^2, x + 2y = c^3$

vi. $x + y = a^2, 2x + y = a^2 + 3b^2, x + 2y = a^2 + c^2$

vii. $x + y = a^2, 2x + y = a^2 + b^2, x + 2y = a^2 + 5c^2$

viii. $x + y = 2a^2, 2x + y = 5a^2 - b^2, x + 2y = 5c^3$

ix. $x + y = z^2, 2x + y = 2z^2 + w^2, x + 2y = 10p^3$

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This communication exhibits different sets of non-zero distinct integer solutions for the system of double equations with four unknowns given by $x - y*z = k*z^3$, $x*y = w^3$

II. METHOD OF ANALYSIS

The dual equations of third degree with four unknowns under consideration is

$$x - y*z = k*z^3 \quad (1)$$

$$x*y = w^3 \quad (2)$$

By scrutiny, observe that (2) is satisfied by

$$x = (s+1)\,\alpha^2 \,, y = (s+1)^2\,\alpha \,, w = (s+1)\,\alpha \quad (3)$$

The substitution of (3) in (1) gives the cubic equation in z as

$$(s+1)\,\alpha^2 - (s+1)^2\, z\,\alpha - k\, z^3 = 0 \quad (4)$$

Treating (4) as a quadratic in α and solving for the same, we get

$$\alpha = \frac{z[(s+1)^2 \pm \sqrt{(s+1)^4 + 4k\,(s+1)\,z}\,]}{2\,(s+1)} \quad (5)$$

The option

$$z = (s+1)\,\sigma \quad (6)$$

in (5) gives

$$\alpha = \frac{\sigma[(s+1)^2 \pm (s+1)\sqrt{(s+1)^2 + 4k\,\sigma}\,]}{2} \quad (7)$$

Again, taking

$$\sigma = (s+1)^2\,(kt^2 \pm t) \quad (8)$$

in (7) , we have

$$\begin{aligned}
\alpha &= \frac{1}{2}(s+1)^4\,(kt^2 \pm t)[1 \pm (2kt \pm 1)] \\
&= \frac{1}{2}(s+1)^4\,(kt^2 + t)[1 \pm (2kt+1)]\,, \frac{1}{2}(s+1)^4\,(kt^2 - t)[1 \pm (2kt-1)] \\
&= (s+1)^4\, t\,(kt+1)^2 \,, -(s+1)^4\, kt^2\,(kt+1), -(s+1)^4\, kt^2\,(kt-1), -(s+1)^4\, t\,(kt-1)^2
\end{aligned} \quad (9)$$

From (9), (8), (6) and (3), we have the following four sets of integer solutions to (1):

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Set 1

$$x = x(k,s,t) = (s+1)^9\, t^2\, (kt+1)^4\,,$$
$$y = y(k,s,t) = (s+1)^6\, t\,(kt+1)^2\,,$$
$$z = z(k,s,t) = (s+1)^3\, t\,(kt+1)\,,$$
$$w = w(k,s,t) = (s+1)^5\, t\,(kt+1)^2\,.$$

Set 2

$$x = x(k,s,t) = (s+1)^9\, k^2\, t^4\, (kt+1)^4\,,$$
$$y = y(k,s,t) = -(s+1)^6\, k\,t^2\,(kt+1)\,,$$
$$z = z(k,s,t) = (s+1)^3\, t\,(kt+1)\,,$$
$$w = w(k,s,t) = -(s+1)^5\, k\,t^2\,(kt+1)\,.$$

Set 3

$$x = x(k,s,t) = (s+1)^9\, k^2\, t^4\, (kt-1)^2\,,$$
$$y = y(k,s,t) = -(s+1)^6\, k\,t^2\,(kt-1)\,,$$
$$z = z(k,s,t) = (s+1)^3\, t\,(kt-1)\,,$$
$$w = w(k,s,t) = -(s+1)^5\, k\,t^2\,(kt-1)\,.$$

Set 4

$$x = x(k,s,t) = (s+1)^9\, t^2\, (kt-1)^4\,,$$
$$y = y(k,s,t) = -(s+1)^6\, t\,(kt-1)^2\,,$$
$$z = z(k,s,t) = (s+1)^3\, t\,(kt-1)\,,$$
$$w = w(k,s,t) = -(s+1)^5\, t\,(kt-1)^2\,.$$

Formulation of Second order Ramanujan numbers:

From each of the integer solutions presented above, one can generate Second order Ramanujan numbers with base numbers as real integers.

Illustration

In Set 1, consider

$$x = x(5,0,1) = 1296$$
$$= 2*648 = 4*324 = 6*216 = 8*162 = 12*108$$
$$= \quad T_1 \quad = \quad T_2 \quad = \quad T_3 \quad = T_4 \quad = \quad T_5 \quad (say)$$

Now,

$$T_1 = T_2 \Rightarrow 325^2 - 323^2 = 164^2 - 160^2$$
$$\Rightarrow 325^2 + 160^2 = 323^2 + 164^2 = 131225$$
$$T_1 = T_3 \Rightarrow 325^2 - 323^2 = 111^2 - 105^2$$
$$\Rightarrow 325^2 + 105^2 = 323^2 + 111^2 = 116650$$

$$T_1 = T_4 \Rightarrow 325^2 - 323^2 = 85^2 - 77^2$$
$$\Rightarrow 325^2 + 77^2 = 323^2 + 85^2 = 111554$$
$$T_1 = T_5 \Rightarrow 325^2 - 323^2 = 60^2 - 48^2$$
$$\Rightarrow 325^2 + 48^2 = 323^2 + 60^2 = 107929$$
$$T_2 = T_3 \Rightarrow 164^2 - 160^2 = 111^2 - 105^2$$
$$\Rightarrow 164^2 + 105^2 = 111^2 + 160^2 = 37921$$
$$T_2 = T_4 \Rightarrow 164^2 - 160^2 = 85^2 - 77^2$$
$$\Rightarrow 164^2 + 77^2 = 85^2 + 160^2 = 32825$$
$$T_2 = T_5 \Rightarrow 164^2 - 160^2 = 60^2 - 48^2$$
$$\Rightarrow 164^2 + 48^2 = 60^2 + 160^2 = 29200$$
$$T_3 = T_4 \Rightarrow 111^2 - 105^2 = 85^2 - 77^2$$
$$\Rightarrow 111^2 + 77^2 = 85^2 + 105^2 = 18250$$
$$T_3 = T_5 \Rightarrow 111^2 - 105^2 = 60^2 - 48^2$$
$$\Rightarrow 111^2 + 48^2 = 60^2 + 105^2 = 14625$$
$$T_4 = T_5 \Rightarrow 85^2 - 77^2 = 60^2 - 48^2$$
$$\Rightarrow 85^2 + 48^2 = 60^2 + 77^2 = 9529$$

Thus, 131225, 116650, 111554, 107929, 37921, 32825, 29200, 18250, 14625 and 9529 are Second order Ramanujan numbers.

It is worth to mention that, one may also consider

$$x = x(5,0,1) = 1296$$
$$= 1*1296 = 3*432 = 27*48$$
$$= f_1 = f_2 = f_3 \quad (say)$$

Now, from $f_1 = f_2$ we get

$$(1296+1)^2 + (432-3)^2 = (1296-1)^2 + (432+3)^2 = 1866250$$

From $f_1 = f_3$ we get

$$(1296+1)^2 + (48-27)^2 = (1296-1)^2 + (432+27)^2 = 1682650$$

From $f_2 = f_3$ we get

$$(432+3)^2 + (48-27)^2 = (432-3)^2 + (48+27)^2 = 189666$$

Thus, 1866250, 1682650 and 189666 are Second order Ramanujan numbers.

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III. CONCLUSION

In this paper, the given system of non-linear non-homogeneous double equations of degree three with four unknowns has been reduced to a single non-homogeneous cubic equation with two unknowns, for which integer solutions can be found successfully, through an elegant non-linear substitutions.

It is worth to mention that the system of non-linear double equations with multiple variables can be reduced to a solvable non-homogeneous cubic equation with lesser variables.

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